2203080301030002
Examination March - 2024
MASTER OF SCIENCE (MATHEMATICS) (FIRST SEMESTER)
TOPOLOGY

[**Time: Three Hours**] **[Max. Marks:70]**

Instructions:	Seat No:
1. Fill up strictly the following details on your answer book a. Name of the Examination: **MASTER OF SCIENCE (MATHEMATICS) (FIRST SEMESTER)** b. Name of the Subject: **TOPOLOGY** c. Subject Code No: **2203080301030002** 2. Sketch neat and labelled diagram wherever necessary. 3. Figures to the right indicate full marks of the question. 4. All questions are compulsory. 5. Follow usual notations and conventions.	Student's Signature

Q.1 Attempt Any Two: 14

1) For a non-empty set X, let x be a point in X and let $\mathcal{T}_x$ consist of empty set $\emptyset$ and all those subsets of X which contains x. Show that $\mathcal{T}_x$ is a topology on X.
2) Let X be the topological space and A be the subset of X then prove that:
   i) $A \cup D(A) = \bar{A}$
   ii) $A\ is\ closed \iff D(A) \subseteq A$
   iii) $Bd(A) = \emptyset \iff A$ is both open and closed
3) Let X be a topological space and Y be a subspace of X. Then show that subspace S of Y is closed in Y if and only if there is a closed set F in X such that $S = F \cap Y$.

Q.2 Attempt Any Two: 14

1) Let X be a second countable space. Then prove that any open base for X has a countable subclass which is also an open base.
2) Show that:
   i) $Int\ (A') = (A)'$, for every subset A of a topological space;
   ii) Boundary of a closed set A of a topological space, is no-where dense.
3) Prove that every separable metric space is second countable.

Q.3 Attempt Any Two: 14

1) Show that any continuous image of a compact space is compact.
2) Show that a continuous real function f defined on a compact space X attains its infimum and its supremum in the following sense: if $a = inf\{f(x): x \in X\}$ and $b = sup\,\{f(x): x \in X\}$, then there exist points $x_1 and\ x_2\ in\ X\ such\ that\ f(x_1) = a\ and\ f(x_2) = b$.
3) Show that a topological space is compact ⟺ every class of closed sets with the finite intersection property has non-empty intersection.

Q.4 Attempt Any Two: 14

1) Define T_1-Space. Prove that a topological space is T_1-space if and only if each point is a closed set.
2) If f is a continuous mapping of a topological space X into a Hausdorff space Y, then prove that the graph of f is a closed subset of the product X × Y.
3) Prove that a one-to-one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism.

Q.5 Attempt Any Two: 14

1) Let X be a topological space. Then show that any closed subset of X is the disjoint union of its set of isolated points and its set of limit points, in the sense that it contains these sets, they are disjoint, and it is their union.
2) State and prove Urysohn's Lemma.
3) Show that every metric space Normal.

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